ENGG. MECHANICS (SOLUTION)

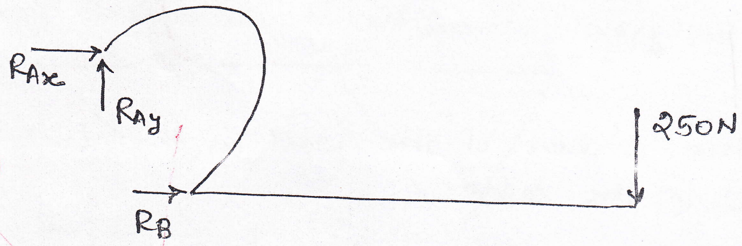
AS-4015

Course 'A'

I Sem.

Unit - I

Ans. 3.



2

$R_B + R_{Ay} = 0$

$R_{Ay} = 250\,N$

$R_B \times 5 = 250 \times 25$

$\boxed{R_B = 1250\,N}$

$R_{Ax} = -1250\,N$

$R_{Ay} = 250\,N$

$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = 1275\,N$

$\theta = 11.3°$

Ans. 1.

SECTION - A

(i) c, third

(ii) a, only single resultant

(iii) b, $m = 2j - 3$

(iv) $\bar{a}$, 0.03

(v) b, non. R

(vi) b, $3r/8$

(vii) $\bar{a}$, halved

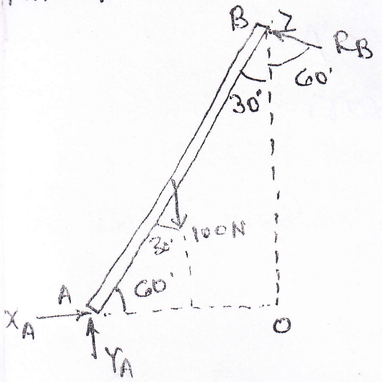
(viii) d, Varignon's th

(ix) b, $0.6r$

(x) b, 2m to the left

Unit - I SECTION — B

Ans. 4.



Considering the free body of the ladder. The various forces acting on the ladder are,

i) Weight of the man acting at C = 100 N

ii) Reaction of the ground at A, having components X_A & Y_A

iii) Reaction of the corner of the wall on the ladder acting normal to the ladder

Writing the equations of equilibrium

$\Sigma F_x = 0$; $\quad X_A - R_B \sin 60° = 0$ —— (i)

$\Sigma F_y = 0$; $\quad Y_A - W - R_B \cos 60° = 0$ —— (ii)

Taking Moment about A, where $AB = OB \sec 30°$

$AB = 4 \times 1.154 = 4.614$ m

$\Sigma M_A = 0$; $\quad R_B(AB) - 100(AD) = 0$

$R_B = 25$ N Ans

$AD = CD \tan 30°$

$= 2 \times 0.577$

$AD = 1.154$ m

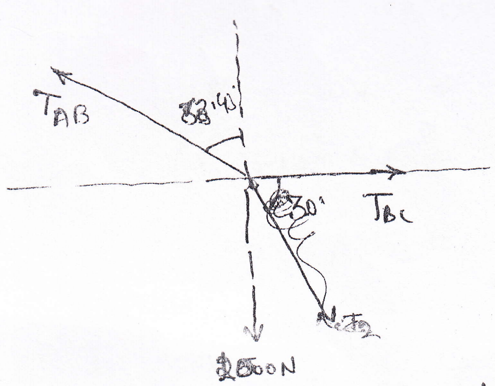
2 (ii)

from (i), $X_A = R_B \sin 60° = 25 \times 0.866 = 21.65$ N Ans

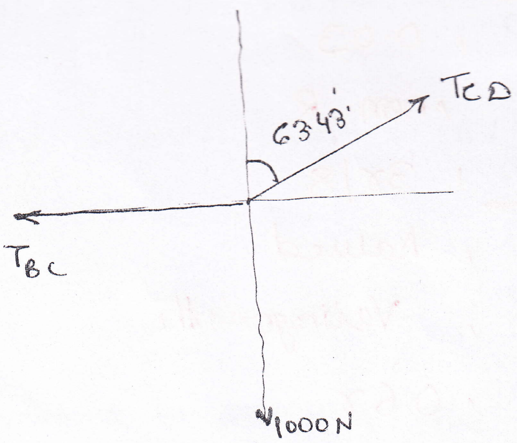
from (ii), $Y_A = W + R_B \cos 60° = 100 + 25 \times 0.5 = 112.5$ N Ans

Ans. 2.

(iii) At point B



At point C



$\tan\theta = \frac{1}{0.5}$

$\theta = 63.43'$

Write equilibrium equation

$\Sigma F_x = 0$; $\quad -T_{AB} \sin 63.43° + T_2 = 0$

$\Sigma F_y = 0$; $\quad T_{AB} \cos 63.43 - 1000 = 0$

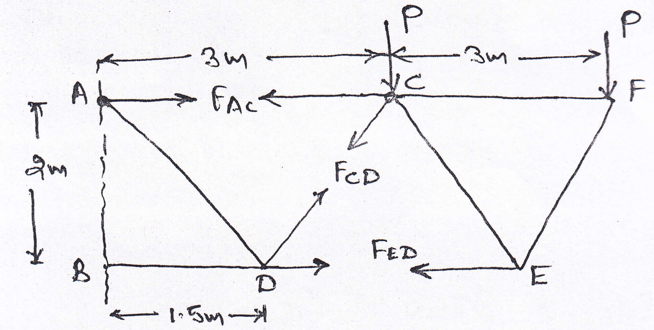
$\Sigma F_x = 0$; $\quad -T_{BC} + T_{CD} \sin 63.43 = 0$

$\Sigma F_y = 0$; $\quad T_{CD} \cos 63.43 - 1000 = 0$

$T_{AB} = 2236$ N $\quad T_{BC} = 2000$ N $\quad T_{CD} = 2236$ N

Unit - II

Ans. 5.



Use Method of Section -
In this case we need not determine the support reactions.

~~The force~~ in section mn cutting the members AC, DC and DF.
Consider the equilibrium of the right hand portion of the truss as shown in figure.

Taking moment about D.

$\Sigma M_D = 0$; $\quad F_{AC} \times 2 - P(1.5) - P(4.5) = 0$

$$F_{AC} = \frac{6P}{2}$$

3(i)

$$F_{AC} = 3P$$

As the force in the member is 3 KN

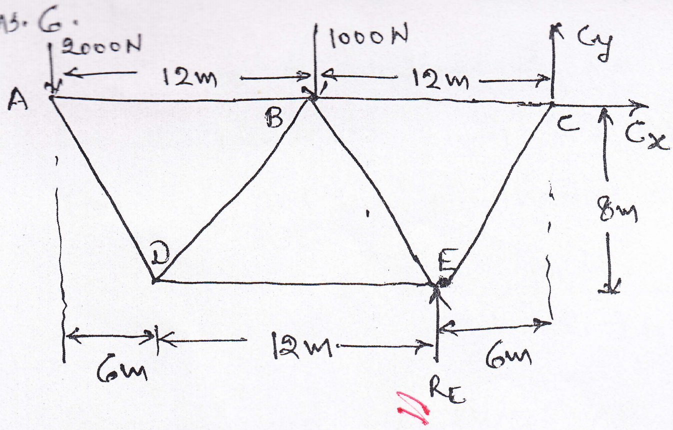
$$F_{AC} = 3\,KN = 3P$$

$$\boxed{P = 1\,KN}$$

OR

Ans. 6.

3(i)



$\Sigma M_C = 0$;

$2000 \times 24 + 1000 \times 12 - R_E \times 6 = 0$

$R_E = 10{,}000\ N$

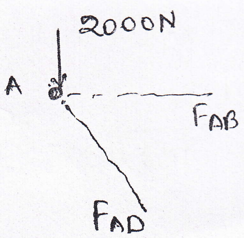
$\Sigma F_x = 0$; $\quad C_x = 0$

$\Sigma F_y = 0$;

$-2000 - 1000 + 10000 + C_y = 0$

$C_y = -7000\ N$

Joint A



$F_{AB} = 1500\,N\ (T)$

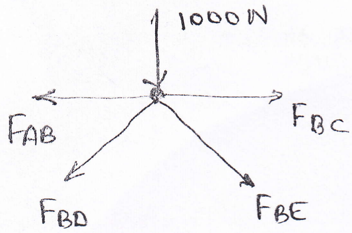
$F_{AD} = 2500\,N\ (C)$

Joint D

$F_{DB} = 2500\,N$ (T)

$F_{DE} = 3000\,N$ (C)

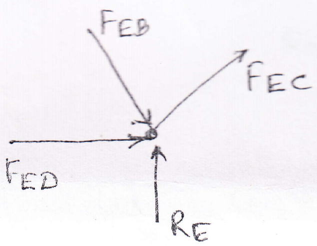
Joint B



$F_{BE} = 3750\,N$ (C)

$F_{BC} = 5250\,N$ (T)

Joint E

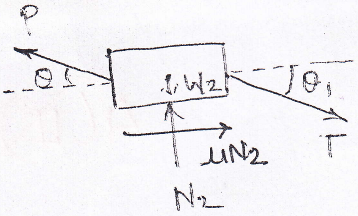


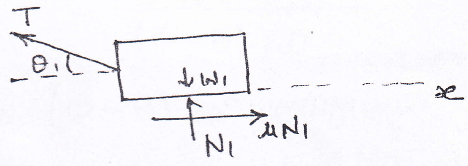
$F_{EC} = 8750\,N$ (C)

Unit - III

Ans. 7.

(4) (i)





Let the force P make an angle θ with the horizontal

Angle θ_1 & θ are not known.

Writing the equations of equilibrium of block W_1

$\Sigma F_x = 0$; $\quad \mu N_1 - T\cos\theta_1 = 0$ ——— (i)

$\Sigma F_y = 0$; $\quad T\sin\theta_1 + N_1 - W_1 = 0$ ——— (ii)

Writing the equations of equilibrium of block W_2

$\Sigma F_x = 0$; $\quad \mu N_2 + T\cos\theta_1 - P\cos\theta = 0$ ——— (iii)

$\Sigma F_y = 0$; $\quad N_2 - W_2 + P\sin\theta - T\sin\theta_1 = 0$ ——— (iv)

There are 4 equations & six unknown ($N_1, N_2, T, P, \theta_1, \theta$).

Let us eliminate T, N_1, N_2 and θ_1

From (i) and (ii) eliminating N_1

$$T = \frac{W_1 \mu}{\mu\sin\theta_1 + \cos\theta_1} \qquad \text{——— (v)}$$

From (iii) and (iv) eliminating N_2

$$T = \frac{P(\cos\theta + \mu\sin\theta) - \mu W_2}{(\mu\sin\theta_1 + \cos\theta_1)} \qquad \text{——— (vi)}$$

Eliminating T from (v) and (vi)

$$\frac{W_1 \mu}{\mu\sin\theta_1 + \cos\theta_1} = \frac{P(\cos\theta + \mu\sin\theta) - \mu W_2}{(\mu\sin\theta_1 + \cos\theta_1)}$$

$$P = \frac{\mu(W_1 + W_2)}{(\cos\theta + \mu\sin\theta)}$$

But $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$$P = \frac{\dfrac{\sin\phi}{\cos\phi}(W_1 + W_2)}{\cos\theta + \dfrac{\sin\phi}{\cos\phi}\sin\theta}$$

$$P = \frac{(W_1 + W_2)\sin\phi}{\cos(\theta - \phi)}$$

4(II)

For P to be minimum $\cos(\theta-\phi)$ should be maximum

$$\cos(\theta - \phi) = 1 \,, \quad (\theta - \phi) = 0 \;; \quad \theta = \phi$$

$P_{min} = (W_1 + W_2)\sin\theta$ and act at an angle $\theta = \phi$ with the horizontal.

Ans. Q. (i) Law of Dry Friction –

4(III)

1. The total friction that can be developed is independent of the magnitude of the area in contact.
2. The total friction that can be developed is proportional to the normal force transmitted across the surface of contact. $F \propto N \Rightarrow F = \mu N$
3. For low velocities, total amount of frictional force that can be developed is practically independent of velocity. But, the force necessary to start the motion is greater than that necessary to maintain the motion.

ii) Self Locking –

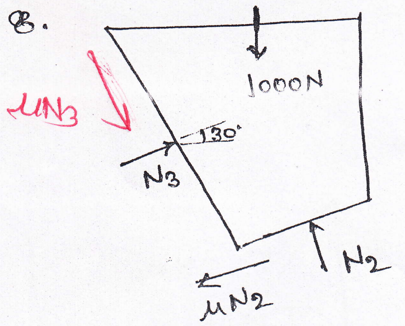
When the load on the screw remains in place even after the effort is removed (that is when effort is zero) it is called self locking machine. It can be shown that for the screw to be self locking $\phi > \theta$.

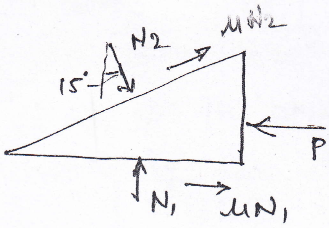
ϕ = Angle of friction

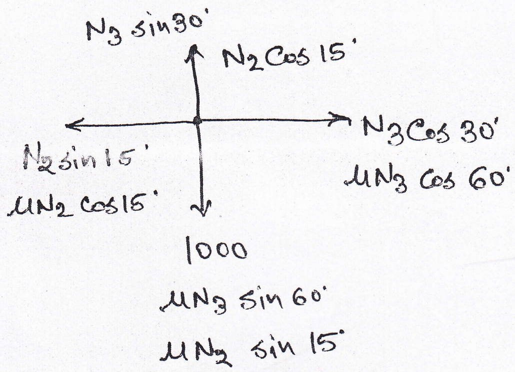
θ = Helix angle

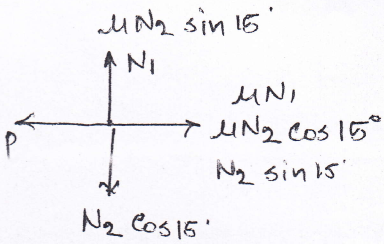
Ans. 8.

4(II)









$\Sigma F_x = 0$;

$$N_3 \cos 30 + \mu N_3 \cos 60 = N_2 \sin 15° + \mu N_2 \cos 15°$$

$$\boxed{N_2 = 2.137\, N_3} \quad \text{——— (1)}$$

$\Sigma F_y = 0$;

$$N_3 \sin 30° + N_2 \cos 15° = 1000 + \mu N_3 \sin 60° + \mu N_2 \sin 15°$$

$$0.3267\, N_3 = 1000 - (0.9141)\, N_2 \quad \text{——— (2)}$$

from (1) & (2)

$$0.3267\, N_3 = 1000 - 0.9141 \times 2.137\, N_3$$

$$\boxed{N_3 = 538.55\, N}$$

$$\boxed{N_2 = 937.17\, N}$$

Wedge -

$$N_2 \cos 15^\circ = \mu N_2 \sin 15^\circ + N_1$$

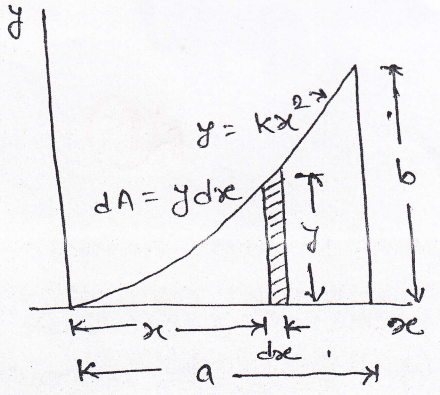
$$\boxed{N_1 = 856.72\ N}$$

$$P = \mu N_2 \cos 15^\circ + N_2 \sin 15^\circ + \mu N_1$$

$$\boxed{P = 595\ N}$$

Unit - IV

Ans. 10. 4(I)



The equation of parabola is

$$y = kx^2 \qquad \text{—— (i)}$$

The value of the constant k is determined by substituting the co-ordinates (a,b) of a point on the curve in the equation (i)

$$b = ka^2 \quad \Rightarrow \quad k = b/a^2$$

The equation of the curve can be written as

$$y = \frac{b}{a^2}x^2 \quad \Rightarrow \quad x = \frac{a}{\sqrt{b}}\sqrt{y}$$

By Vertical strip – Consider a vertical differential element (or strip) of height y and width dx.

Area of the strip $dA = y\,dx$

Distance of the centroid of the strip from the y-axis $= x$

$$x_c = \frac{\int x\,dA}{\int dA} = \frac{\int x\cdot y\,dx}{\int y\,dx} = \frac{\int_0^a x\left(b/a^2\, x^2\right)dx}{\int_0^a b/a^2\, x^2\,dx}$$

$$x_c = \frac{\left[\frac{b}{a^2}\cdot\frac{x^4}{4}\right]_0^a}{\left[\frac{b}{a^2}\cdot\frac{x^3}{3}\right]_0^a}$$

$$\boxed{x_c = \frac{3}{4}a}$$

$$y_c = \frac{\int y\,dA}{\int dA}$$

Distance of the centroid of the strip from the x-axis $= y/2$

$$y_c = \frac{\int y/2\cdot y\,dx}{\int y\,dx} = \frac{\int_0^a 1/2\, y^2\,dx}{\int_0^a y\,dx} = \frac{\int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2dx}{\int_0^a b/a^2\, x^2\,dx}$$

$$y_c = \frac{\left[\frac{b^2}{2a^4}\cdot\frac{x^5}{5}\right]_0^a}{\left[\frac{b}{a^2}\cdot\frac{x^3}{3}\right]_0^a}$$

$$\boxed{y_c = \frac{3}{10}b}$$

Ans. 11.

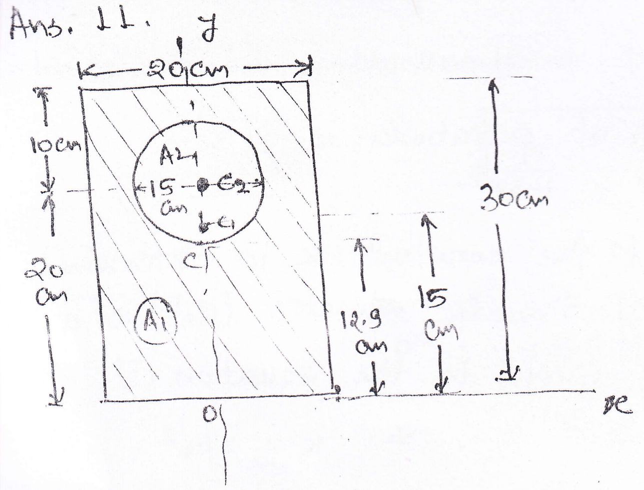


Figure	Area (cm^2)	x-coordinate of the Centroid (cm)	y-coordinate of the centroid (cm)
Rectangle	$A_1 = 20 \times 30 = 600$	$x_1 = 0$	$y_1 = 15$
Circle Removed	$A_2 = \frac{\pi}{4} 15^2 = 176.7$	$x_2 = 0$	$y_2 = 20$

$$y_c = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{600 \times 15 - 176.7 \times 20}{600 - 176.7}$$

$y_c = 12.9$ cm from the x-axis or from the bottom edge

The axis x' drawn parallel to the x-axis and passing through the centroid c of the composite area is called as the centroidal x-axis of the composite area.

Area A_1: M.I. of the area A_1 about it's centroidal x-axis

$$(\bar{I}_x)_1 = \frac{20 \times 30^3}{12} = 45000 \text{ cm}^4.$$

M.I. of the area A_1 about the centroidal x-axis of the composite area

$$I_{x'} = (\bar{I}_x)_1 + A_1 d^2 \quad \text{(By parallel-axis theorem)}$$

$$= 45000 + 600 \times 2.1^2$$

$$I_{x'} = 47646 \text{ cm}^4$$

Area A_2: M.I. of the area A_2 about it's centroidal x-axis

$$(\bar{I}_x)_2 = \frac{\pi}{64}(15)^4 = 2485 \text{ cm}^4$$

M.I. of the area A_2 about the centroidal x-axis of the composite area

$$(I_x')_2 = (I_x)_2 + A_2 d^2 \quad \text{(By Parallel Axis theorem)}$$

$$= 2485 + 176.7\,(7.1)^2$$

$$(I_x')_2 = 11392 \text{ cm}^4$$

Composite Area : Moment of inertia of the composite area about the centroidal x-axis

$$I_x' = (I_x')_1 - (I_x')_2 = 47646 - 11392$$

$$\boxed{I_x' = 36254 \text{ cm}^4}$$

Ans. 12.

5(III)

$$I_x' = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_x = \frac{bd^3}{12} = \frac{a^4}{12} \qquad I_{xy} = a^2 \times (0) = 0$$

$$I_y = \frac{bd^3}{12} = \frac{a^4}{3}$$

$$I_x' = \frac{a^4/12 + a^4/3}{2} + \frac{a^4/12 - a^4/3}{2}\cos(2 \times 45)$$

$$= \frac{5}{24}a^4 - \frac{a^4}{8}(0)$$

$$\boxed{(I_x') = \frac{5a^4}{24}}$$

$$I_y' = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$\boxed{I_y' = \frac{5a^4}{24}}$$

$$I_{x'y'} = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos(2\theta)$$

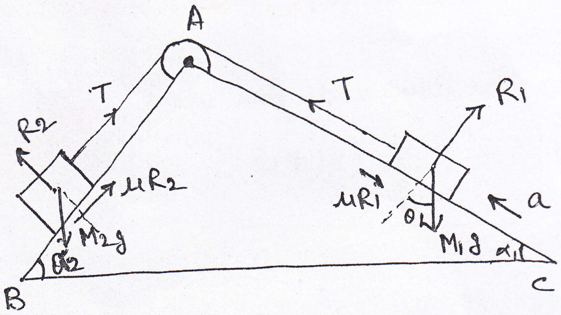
$$= \left(\frac{\frac{a^4}{12} - \frac{a^4}{4}}{2}\right) \sin(90^\circ) + 0$$

$$\boxed{I_{x'y'} = -\frac{a^4}{8}}$$

Unit - V

(6)

Ans. 13.



$\alpha_1 = \theta_1 \qquad \alpha_2 = \theta_2$

Let the acceleration of the mass M_2 parallel to the inclined plane be a. The acceleration of M_1 is same in magnitude.

Motion of Mass M_2: Force acting on the mass M_2 are weight $M_2 g$, tension T, normal reaction R_2 and friction force μR_2.

Writing the equation of motion:

$\Sigma f_x = m a_x$; (along the plane) $\qquad M_2 a = M_2 g \sin\theta_2 - \mu R_2 - T \qquad (a_x = a)$

$\Sigma f_y = m a_y$; (Normal to the plane) $\qquad 0 = R_2 - M_2 g \cos\theta_2$

Eliminating R_2 from the above

$$M_2 a = M_2 g \sin\theta_2 - \mu (M_2 g \cos\theta_2) - T \qquad \text{--- (1)}$$

Motion of Mass M_1 - Force acting on the mass M_1 are weight $M_1 g$, tension T, normal reaction R_1 and friction force μR_1

Writing the equation of motion

$\Sigma f_x = m a_x$: (along the plane) $\qquad M_1 a = T - M_1 g \sin\theta_1 - \mu R_1 \qquad (a_x = a)$

$\Sigma f_y = m a_y$: $\qquad 0 = R_1 - M_1 g \cos\theta_1$

Eliminating R_1 from the above equation

$$M_1 a = T - M_1 g \sin\theta_1 - \mu (M_1 g \cos\theta_1) \qquad \text{--- (2)}$$

Eliminating T from (1) & (2)

$$M_2 g \sin\theta_2 - \mu M_2 g \cos\theta_2 - M_2 a = M_1 a + M_1 g \sin\theta_1 + \mu M_1 g \cos\theta_1$$

$$a (M_1 + M_2) = g [M_2 (\sin\theta_2 - \mu \cos\theta_2) - M_2 (\sin\theta_1 + \mu \cos\theta_1)]$$

$$\boxed{a = \frac{g}{M_1 + M_2} [M_2 (\sin\theta_2 - \mu \cos\theta_2) - M_1 (\sin\theta_1 + \mu \cos\theta_1)]}$$

Ans. 14. Initial velocity of the trolley when the man is standing $= V$

Mass of the trolley $= W/g$

Mass of the man $= w/g$

6 OR

Initial momentum of the system of the trolley and the man

$$= \frac{W}{g} V + \frac{w}{g} V = \frac{V}{g}(W+w)$$

After the man runs,

Let ΔV be the increase in the velocity of the trolley when the man runs and then jumps off.

Final velocity of the trolley $= V + \Delta V$

Final momentum of the trolley $= \frac{W}{g}(V+\Delta V)$

The velocity of man with respect to the trolley $= v$

Absolute velocity of the man = Velocity of man relative to the trolley + Velocity of the trolley

$$= -v + (V + \Delta V)$$

Final momentum of the man

$$= \frac{w}{g}[-v + (V + \Delta V)]$$

The momentum of the man and the trolley is conserved as no external force is acting

$$\frac{(W+w)V}{g} = \frac{W}{g}(V+\Delta V) + \frac{w}{g}[-v + (V+\Delta V)]$$

$$WV + wV = WV + W(\Delta V) - wv + wV + w(\Delta V)$$

$$(W+w)\Delta V = wv$$

$$\Delta V = \frac{wv}{W+w}$$

In terms of the corresponding masses m and M' of the man and the trolley

$$\boxed{\Delta V = \frac{mV}{M+m}}$$

It is importance to note here the increase ΔV in the velocity of the trolley is independent of the initial velocity V of the trolley.